In Exercises 53–56, use a symbolic differentiation utility to find the fifth-degree Taylor polynomial (centered at c) of the function. Graph the function and the polynomial. Use the graph to obtain the largest interval on which the polynomial is a reasonable approximation of the function.

53. $f(x) = x\cos 2x, \quad c = 0$

54. $f(x) = \sin\frac{x}{2}\ln(1 + x), \quad c = 0$

55. $g(x) = \sqrt{x}\ln x, \quad c = 1$

56. $h(x) = \sqrt[3]{x}\arctan x, \quad c = 1$

57. *Projectile Motion* A projectile fired from the ground follows the trajectory given by

$$y = \left(\tan\theta - \frac{g}{kv_0\cos\theta}\right)x - \frac{g}{k^2}\ln\left(1 - \frac{kx}{v_0\cos\theta}\right)$$

where v_0 is the initial speed, θ is the angle of projection, g is the acceleration due to gravity, and k is the drag factor caused by air resistance. Using the power series representation

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \qquad -1 < x < 1$$

verify that the trajectory can be rewritten as

$$y = (\tan\theta)x + \frac{gx^2}{2v_0^2\cos^2\theta} + \frac{kgx^3}{3v_0^3\cos^3\theta} + \frac{k^2gx^4}{4v_0^4\cos^4\theta} + \cdots$$

58. *Projectile Motion* Use the result of Exercise 57 to determine the series for the path of a projectile projected from ground level at an angle of $\theta = 60°$ with an initial speed of $v_0 = 64$ feet per second and a drag factor of $k = \frac{1}{16}$.

59. Consider the function f defined by

$$f(x) = \begin{cases} e^{-1/x^2}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

a. Sketch a graph of the function.
b. Use the alternative form of the definition of the derivative (Section 2.1) and L'Hôpital's Rule to show that $f'(0) = 0$. [By continuing this process, it can be shown that $f^{(n)}(0) = 0$ for $n > 1$.]
c. Using the result of part b, find the Maclaurin series for f. Does the series converge to f?

60. Sketch the graph of

$$f(x) = \begin{cases} 0, & x < -\pi/2 \\ \cos x, & -\pi/2 \le x \le \pi/2. \\ 0, & x > \pi/2 \end{cases}$$

Does the Maclaurin series for this function converge to the function? Explain.

61. Find the Maclaurin series for $f(x) = xe^x$. Integrate this series term-by-term over the closed interval $[0, 1]$, and show that

$$1 = \sum_{n=0}^{\infty} \frac{1}{(n + 2)n!}.$$

62. Prove that $\lim_{n\to\infty} \frac{x^n}{n!} = 0$ for any real x.

63. Prove that e is irrational. (*Hint:* Assume that $e = p/q$ is rational (p, q integers) and consider

$$e = 1 + 1 + \frac{1}{2!} + \cdots + \frac{1}{n!} + \cdots.)$$

REVIEW EXERCISES for Chapter 8

↓ CHAPTER 8 REVIEW ↓ ↓ ↓

In Exercises 1 and 2, find the general term of the sequence.

1. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

2. $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \ldots$

In Exercises 3–10, determine the convergence or divergence of the sequence with the given general terms. (b and c are positive real numbers.)

3. $a_n = \dfrac{n + 1}{n^2}$

4. $a_n = \dfrac{1}{\sqrt{n}}$

5. $a_n = \dfrac{n^3}{n^2 + 1}$

6. $a_n = \dfrac{n}{\ln n}$

7. $a_n = \sqrt{n + 1} - \sqrt{n}$

8. $a_n = \left(1 + \dfrac{1}{2n}\right)^n$

9. $a_n = \dfrac{\sin\sqrt{n}}{\sqrt{n}}$

10. $a_n = (b^n + c^n)^{1/n}$

11. *Compound Interest* A deposit of \$5000 is made in an account that earns 8% interest compounded quarterly. The balance in the account after n quarters is

$$A_n = 5000\left(1 + \frac{0.08}{4}\right)^n, \qquad n = 1, 2, 3, \ldots.$$

a. Compute the first eight terms of this sequence.
b. Find the balance in this account after 10 years by computing the 40th term of the sequence.

12. *Depreciation* A company buys a machine for \$120,000. During the next 5 years it will depreciate at the rate of 30% per year. (That is, at the end of each year the depreciated value is 70% of what it was at the beginning of the year.)
a. Find the formula for the nth term of a sequence that gives the value of the machine t full years after it was purchased.
b. Find the depreciated value of the machine at the end of 5 full years.

In Exercises 13–16, find the first five terms of the sequence of partial sums for the series.

13. $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

14. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n}$

15. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)!}$

16. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

In Exercises 17–20, find the sum of the series.

17. $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

18. $\sum_{n=0}^{\infty} \frac{2^{n+2}}{3^n}$

19. $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$

20. $\sum_{n=0}^{\infty} \left[\left(\frac{2}{3}\right)^n - \frac{1}{(n+1)(n+2)}\right]$

In Exercises 21 and 22, express the repeating decimal as the ratio of two integers.

21. $0.\overline{09}$

22. $0.\overline{923076}$

23. *Bouncing Ball* A ball is dropped from a height of 8 feet. Each time it drops h feet, it rebounds $0.7h$ feet. Find the total distance traveled by the ball.

24. *Total Compensation* Suppose you accept a job that pays a salary of $32,000 the first year. During the next 39 years you receive a 5.5% raise each year. What would your total salary be over the 40-year period?

25. *Compound Interest* A deposit of $200 is made at the end of each month for 2 years in an account that pays 6%, compounded continuously. What is the balance in the account at the end of the 2 years?

26. *Compound Interest* A deposit of $100 is made at the end of each month for 10 years in an account that pays 6.5%, compounded monthly. What is the balance in the account at the end of the 10 years?

In Exercises 27–38, determine the convergence or divergence of the series.

27. $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$

28. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

29. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 2n}}$

30. $\sum_{n=1}^{\infty} \frac{n+1}{n(n+2)}$

31. $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

32. $\sum_{n=1}^{\infty} \frac{n!}{e^n}$

33. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln n}$

34. $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1}$

35. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n}\right)$

36. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{2^n}\right)$

37. $\sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)(2n+1)}$

38. $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 5 \cdot 8 \cdots (3n-1)}$

39. *Essay* Use a computer to complete the table for
a. $p = 2$ b. $p = 5$
Write a short paragraph describing and comparing the entries in the tables generated.

N	5	10	20	30	40
$\sum_{n=1}^{N} \frac{1}{n^p}$					
$\int_N^{\infty} \frac{1}{x^p}\,dx$					

40. *Essay* You are told that the terms of a positive series appear to approach 0 very slowly as $n \to \infty$. (In fact, $a_{75} = 0.7$.) If you are given no other information, can you conclude that the series diverges? Support your answer with examples.

In Exercises 41–46, find the interval of convergence of the power series.

41. $\sum_{n=0}^{\infty} \left(\frac{x}{10}\right)^n$

42. $\sum_{n=0}^{\infty} (2x)^n$

43. $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{(n+1)^2}$

44. $\sum_{n=1}^{\infty} \frac{3^n (x-2)^n}{n}$

45. $\sum_{n=0}^{\infty} n!(x-2)^n$

46. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{2^n}$

In Exercises 47–54, find the power series for the function centered at c.

47. $f(x) = \sin x, \quad c = \frac{3\pi}{4}$

48. $f(x) = \cos x, \quad c = -\frac{\pi}{4}$

49. $f(x) = 3^x, \quad c = 0$

50. $f(x) = \csc x, \quad c = \frac{\pi}{2}$
(first three terms)

51. $f(x) = \frac{1}{x}, \quad c = -1$

52. $f(x) = \sqrt{x}, \quad c = 4$

53. $g(x) = \frac{2}{3-x}, \quad c = 0$

54. $h(x) = \frac{1}{(1+x)^3}, \quad c = 0$

55. Determine the first four terms of the Maclaurin series for e^{2x}
a. by using the definition of the Maclaurin series and the formula for the coefficient of the nth term $a_n = f^{(n)}(0)/n!$.
b. by replacing x by $2x$ in the series for e^x.
c. by multiplying the series for e^x by itself because $e^{2x} = e^x \cdot e^x$.

56. Follow the pattern of Exercise 55 to find the first four terms of the series for $\sin 2x$. [*Hint*: $\sin 2x \doteq 2 \sin x \cos x$.]

5. $\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{n+1}}{n+1}$ 7. $\sum_{n=0}^{\infty}\frac{(-1)^n(2x)^{2n+1}}{(2n+1)!}$

9. $1+\frac{x^2}{2!}+\frac{5x^4}{4!}+\cdots$

11. $\sum_{n=0}^{\infty} (-1)^n(n+1)x^n$

13. $\frac{1}{2}\left[1+\sum_{n=1}^{\infty} \frac{(-1)^n 1\cdot3\cdot5\cdots(2n-1)x^{2n}}{2^{3n}n!}\right]$

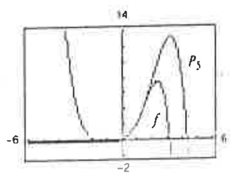
15. $1+\frac{x^2}{2}+\sum_{n=2}^{\infty}\frac{(-1)^{n+1}1\cdot3\cdot5\cdots(2n-3)x^{2n}}{2^n n!}$

17. $1+\frac{x^2}{2}+\frac{x^4}{2^2 2!}+\frac{x^6}{2^3 3!}+\cdots$ 19. $\sum_{n=0}^{\infty} \frac{(-1)^n(2x)^{2n+1}}{(2n+1)!}$

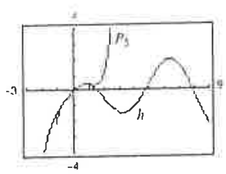
21. $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{(2n)!}$ 23. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$ 25. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

27. $\frac{1}{2}\left[1+\sum_{n=0}^{\infty}\frac{(-1)^n(2x)^{2n}}{(2n)!}\right]$ 29. $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$

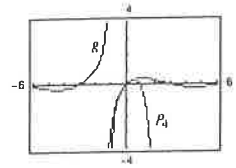
31. $P_5(x)=x+x^2+\frac{1}{3}x^3-\frac{1}{30}x^5+\cdots$



33. $P_5(x)=x-\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{3}{40}x^5+\cdots$



35. $P_4(x)=x-x^2+\frac{5}{6}x^3-\frac{5}{6}x^4+\cdots$

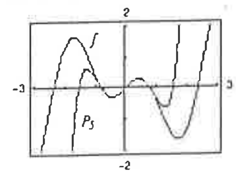


37. $\sum_{n=0}^{\infty}\frac{(-1)^{(n+1)}x^{2n+3}}{(2n+3)(n+1)!}$ 39. 0.8415 41. 0.6931

43. 0 45. 0.9461 47. 0.5312 49. 0.2010

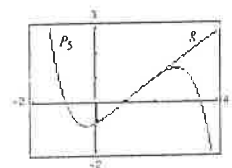
51. 0.3413

53. $P_5(x)=x-2x^3+\frac{2}{3}x^5$
$\left[-\frac{3}{4},\frac{3}{4}\right]$

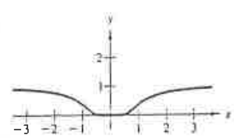


55. $P_5(x)=(x-1)-\frac{1}{24}(x-1)^3+\frac{1}{24}(x-1)^4-\frac{71}{1920}(x-1)^5$

$\left[\frac{1}{4},2\right]$



59. a.



c. $\sum_{n=0}^{\infty} 0x^n = 0 \neq f(x)$

Answers ↓ ↓ ↓

Review Exercises for Chapter 8

1. $a_n=\frac{1}{n!}$ 3. Converges to 0 5. Diverges

7. Converges to 0 9. Converges to 0

11. a.

n	1	2	3	4	5	6	7	8
A_n	\$5100.00	\$5202.00	\$5306.04	\$5412.16	\$5520.40	\$5630.81	\$5743.43	\$5858.30

b. \$11,040.20

13. 1, 2.5, 4.75, 8.125, 13.3875
15. 0.5, 0.45833, 0.45972, 0.45970, 0.45970
17. 3   19. $\frac{1}{2}$ 21. $\frac{1}{11}$ 23. $45\frac{1}{3}$ ft
25. \$5087.14   27. Diverges   29. Converges
31. Converges   33. Diverges   35. Diverges
37. Converges

39. a.

N	5	10	20	30	40
$\sum_{n=1}^{N} \frac{1}{n^p}$	1.4636	1.5498	1.5962	1.6122	1.6202
$\int_N^\infty \frac{1}{x^p}\,dx$	0.2000	0.1000	0.0500	0.0333	0.0250

b.

N	5	10	20	30	40
$\sum_{n=1}^{N} \frac{1}{n^p}$	1.0367	1.0369	1.0369	1.0369	1.0369
$\int_N^\infty \frac{1}{x^p}\,dx$	0.0004	0.0000	0.0000	0.0000	0.0000

The series of part b converges more rapidly. This is evident from the integrals which give the remainders of the partial sums.

41. $(-10, 10)$ 43. $[1, 3]$ 45. Converges only at $x = 2$

47. $\frac{\sqrt{2}}{2}\sum_{n=0}^{\infty}\frac{(-1)^{n(n+1)/2}}{n!}\left(x - \frac{3\pi}{4}\right)^n$ 49. $\sum_{n=0}^{\infty}\frac{(x \ln 3)^n}{n!}$

51. $-\sum_{n=0}^{\infty}(x-1)^n$ 53. $\sum_{n=0}^{\infty}\frac{2}{3}\left(\frac{x}{3}\right)^n$

55. $1 + 2x + 2x^2 + \frac{4}{3}x^3$ 57. $f(x) = \frac{3}{3 - 2x}$

$\left(-\frac{3}{2}, \frac{3}{2}\right)$

59. $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$ 61. $\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{(n+1)^2}$

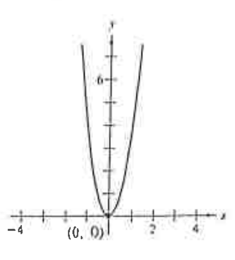
63. 0   65. 0.996   67. 0.560
69. a. 4   b. 6   c. 5   d. 10

CHAPTER 9

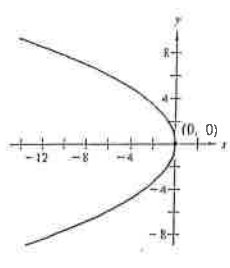
Section 9.1 (*page 640*)

1. e   3. a   5. d

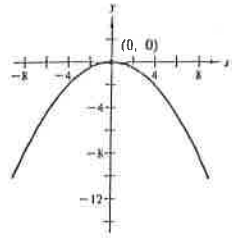
7. Vertex: $(0, 0)$
Focus: $(0, \frac{1}{16})$
Directrix: $y = -\frac{1}{16}$



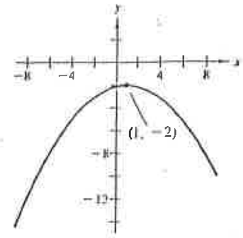
9. Vertex: $(0, 0)$
Focus: $(-\frac{3}{2}, 0)$
Directrix: $x = \frac{3}{2}$



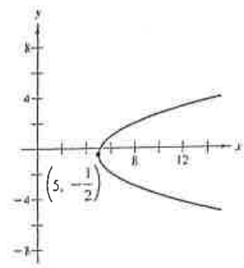
11. Vertex: $(0, 0)$
Focus: $(0, -2)$
Directrix: $y = 2$



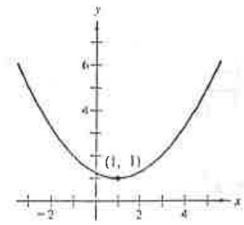
13. Vertex: $(1, -2)$
Focus: $(1, -4)$
Directrix: $y = 0$



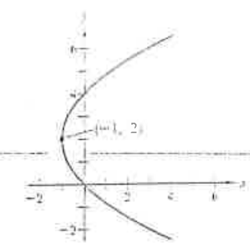
15. Vertex: $(5, -\frac{1}{2})$
Focus: $(\frac{11}{2}, -\frac{1}{2})$
Directrix: $x = \frac{9}{2}$



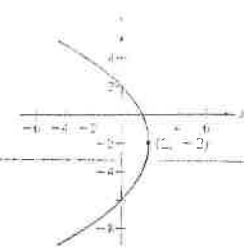
17. Vertex: $(1, 1)$
Focus: $(1, 2)$
Directrix: $y = 0$



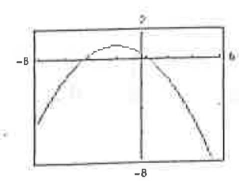
19. Vertex: $(-1, 2)$
Focus: $(0, 2)$
Directrix: $x = -2$



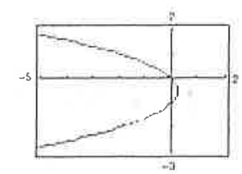
21. Vertex: $(2, -2)$
Focus: $(0, -2)$
Directrix: $x = 4$



23. Vertex: $(-2, 1)$
Focus: $(-2, -\frac{1}{2})$
Directrix: $y = \frac{5}{2}$



25. Vertex: $(\frac{1}{4}, -\frac{1}{2})$
Focus: $(0, -\frac{1}{2})$
Directrix: $x = \frac{1}{2}$



27. $x^2 + 6y = 0$ 29. $y^2 - 4y + 8x - 20 = 0$
31. $x^2 + 24y + 96 = 0$ 33. $y^2 - 8x - 4y + 4 = 0$
35. $x^2 + y - 4 = 0$ 37. $5x^2 - 14x - 3y + 9 = 0$
39. $(x - h)^2 = -8(y + 1)$ 41. $3x - 2y^2 = 0$
43. $\frac{9}{4}$ ft   45. $4x - y - 8 = 0$ 47. $y = 2ax_0x - ax_0^2$
49. Tangent lines: $2x + y - 1 = 0$
$2x - 4y - 1 = 0$
Point of intersection: $(\frac{1}{2}, 0)$
(on the directrix)